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INTEGRAL REPRESENTATION DISCONTINUOUS SOLUTION OF THE PROBLEM OF BENDING OF ANISOTROPIC PLATES

Leaning on the ratios connecting deflection derivatives as the generalized function, with usual derivatives, the differential equation which right part contains the generalized functions having jumps of a deflection, tilt angles, the moments and generalized shear forces are resived. The solution of the equation is received in the form of convolution of the fundamental decision with the right part. From the found representation the boundary integrated equations (BIE) for the solution of the problem can be received. These BIE can be solved by method of boundary elements.

Key words: *bending, an anisotropic plate, defects, discontinuous solution, generalized function, boundary value problem.*

Introduction

The structures of many machines have plate elements. In these plates can be formed cracks. In addition they may contain thin inserts of other materials. The research of stress-strain state of such plates is an important problem. At the same time, the solution of the corresponding boundary value problems causing serious mathematical difficulties. To solve these problems G.Y. Popov proposed a generalized method of integral transforms [1]. This method was developed in the work of G.A. Morar [2]. S. Crouch proposed a method of discontinuous shifts, alternatively boundary element method (BEM) [3]. The corresponding boundary elements for anisotropic media were obtained in [4, 5]. There used the relationship between ordinary and generalized derivatives of regular generalized functions. This technique is used in this work.

If the boundary of the extended straight portion includes a rigid or articulated fixing, then can be used as $G(x, y, \xi, \eta)$ Green's function obtained in [7, 8].

Formulating and solving problems

Consider the following problem:

$$D_{11}\frac{\partial^4 w}{\partial x^4} + 4D_{16}\frac{\partial^4 w}{\partial x^3 \partial y} + 2(D_{12} + 2D_{66}) \times$$
$$\times \frac{\partial^4 w}{\partial x^2 \partial y^2} + 4D_{26}\frac{\partial^4 w}{\partial x \partial y^3} + D_{22}\frac{\partial^4 w}{\partial y^4} = q(x, y), \quad (1)$$

where $w(x, y)$ – the deflection at the point (x, y); $(x, y) \in B \subset R^2$, B – limited area, l_0 – piecewise smooth boundary of B, $l_i = A_i B_i$ ($i = \overline{1,k}$) – smooth curves lying in B. These curves can be closed, they can coincide ends. The end of one arc may be an interior point of the other. On the line l_0 are given two boundary conditions. Also, two conditions are given by on the lines l_i.

Equation (1) describes the bending of anisotropic plate with rigidity anisotropy D_{11}, D_{12}, D_{16}, D_{26}, D_{22}, D_{66}.

Curves l_i simulate cracks or thin inclusions or reinforcements.

If the curve $l \subset B \setminus \bigcup_{i=0}^{k} l_i$, then are set there the following functions

$$M_n = -\left(D_{11}\frac{\partial^2 w}{\partial x^2} + D_{12}\frac{\partial^2 w}{\partial y^2} + 2D_{16}\frac{\partial^2 w}{\partial x \partial y} \right) n_x^2 -$$
$$-\left(D_{12}\frac{\partial^2 w}{\partial x^2} + D_{22}\frac{\partial^2 w}{\partial y^2} + 2D_{26}\frac{\partial^2 w}{\partial x \partial y} \right) n_y^2 -$$
$$-2\left(D_{16}\frac{\partial^2 w}{\partial x^2} + D_{26}\frac{\partial^2 w}{\partial y^2} + 2D_{66}\frac{\partial^2 w}{\partial x \partial y} \right) n_x n_y ;$$

$$H_{tn} = \left((D_{11}\frac{\partial^2 w}{\partial x^2} + D_{12}\frac{\partial^2 w}{\partial y^2} + 2D_{16}\frac{\partial^2 w}{\partial x \partial y} - D_{12}\frac{\partial^2 w}{\partial x^2} - \right.$$
$$- D_{22}\frac{\partial^2 w}{\partial y^2} - 2D_{26}\frac{\partial^2 w}{\partial x \partial y}) n_x n_y +$$
$$+\left(D_{16}\frac{\partial^2 w}{\partial x^2} + D_{26}\frac{\partial^2 w}{\partial y^2} + 2D_{66}\frac{\partial^2 w}{\partial x \partial y} \right)(n_y^2 - n_x^2) ;$$

$$N_n = -\left(D_{11}\frac{\partial^3 w}{\partial x^3} + 3D_{16}\frac{\partial^3 w}{\partial x^2 \partial y} + (D_{12}+2D_{66})\frac{\partial^3 w}{\partial x \partial y^2} + \right.$$

$$\left. + D_{26}\frac{\partial^3 w}{\partial y^3}\right) n_x - \left(D_{16}\frac{\partial^3 w}{\partial x^3} + (D_{12}+2D_{66})\frac{\partial^3 w}{\partial x^2 \partial y} + \right.$$

$$\left. + 3D_{26}\frac{\partial^3 w}{\partial x \partial y^2} + D_{22}\frac{\partial^3 w}{\partial y^3}\right) n_y ; \; Q_n = N_n + \frac{\partial H_{tn}}{\partial s} ;$$

$$\frac{\partial w}{\partial n} = \frac{\partial w}{\partial x} n_x + \frac{\partial w}{\partial y} n_y ; \; \frac{\partial w}{\partial \tau} = -\frac{\partial w}{\partial x} n_y + \frac{\partial w}{\partial y} n_x .$$

There M_n – bending moment, H_{tn} – twisting moment, N_n – shear force, Q_n – generalized shear force, $\frac{\partial w}{\partial n}$ – normal angle of inclination, $\frac{\partial w}{\partial \tau}$ – tangent angle of inclination. The tangent vector $\bar{\tau}$ is chosen such that the three vectors $\bar{n}$, $\bar{\tau}$, $\bar{k}$ form a right-handed vectors.

Let us introduce the notation:

$n^{(0)}$ – unit normal vector to the line l_0 outward region B;

$n^{(i)} = (n_x^{(i)}, n_y^{(i)})$ – arbitrarily chosen unit normal vector to l_i, $i = \overline{1,k}$;

$\tau^{(0)}$ – unit tangent vector to l_0; $\tau^{(i)} = (\tau_x^{(i)}, \tau_y^{(i)})$ – unit tangent vector to l_i;

$n_x^{(i)} = \cos(n^{(i)}, x)$, $n_y^{(i)} = \cos(n^{(i)}, y)$, $\tau_x^{(i)} = -n_y^{(i)}$, $\tau_y^{(i)} = n_x^{(i)}$ $i = \overline{0,k}$.

Denote the $D^\alpha w(x,y)$ – derivative of $w(x,y) \in D'(R^2)$; $\{D^\alpha w(x,y)\}$ – ordinary derivative $w(x,y)$; $(x,y) \in R^2 \setminus \bigcup_{i=0}^{k} l_i$.

Using the relationship between $D^\alpha w(x,y)$ and $\{D^\alpha w(x,y)\}$, we get:

$$\frac{\partial^4 w}{\partial x^4} = \left\{\frac{\partial^4 w}{\partial x^4}\right\} + \left\{\frac{\partial^3 w}{\partial x^3}\right\} n_x^{(0)} \delta(l_0) +$$

$$+ \frac{\partial^2}{\partial x^2}\left(\left\{\frac{\partial w}{\partial x}\right\} n_x^{(0)} \delta(l_0)\right) + \frac{\partial^3}{\partial x^3}\left(w n_x^{(0)} \delta(l_0)\right) +$$

$$+ \sum_{i=1}^{k}\left(\left[\frac{\partial^3 w}{\partial x^3}\right] n_x^{(i)} \delta(l_i) + \frac{\partial}{\partial x}\left(\left[\frac{\partial^2 w}{\partial x^2}\right] n_x^{(i)} \delta(l_i)\right) + \right.$$

$$\left. + \frac{\partial^2}{\partial x^2}\left(\left[\frac{\partial w}{\partial x}\right] n_x^{(i)} \delta(l_i)\right) + \frac{\partial^3}{\partial x^3}\left([w] n_x^{(i)} \delta(l_i)\right)\right);$$

Next $\frac{\partial^4 w}{\partial y^4}$ is obtained from $\frac{\partial^4 w}{\partial x^4}$ by replacing the variable x on y.

For $\frac{\partial^4 w}{\partial x^3 \partial y}$ have the four options representations.

The first option.

$$\frac{\partial^4 w}{\partial x^3 \partial y} = \left\{\frac{\partial^4 w}{\partial x^3 \partial y}\right\} + \left\{\frac{\partial^3 w}{\partial x^2 \partial y}\right\} n_x^{(0)} \delta(l_0) +$$

$$+ \frac{\partial}{\partial x}\left(\left\{\frac{\partial^2 w}{\partial x \partial y}\right\} n_x^{(0)} \delta(l_0) + \frac{\partial^2}{\partial x^2}\left(\left\{\frac{\partial w}{\partial x}\right\} n_x^{(0)} \delta(l_0)\right) + \right.$$

$$+ \frac{\partial^3}{\partial x^3}\left(w n_y^{(0)} \delta(l_0)\right) + \sum_{i=1}^{k}\left(\left[\frac{\partial^3 w}{\partial x^2 \partial y}\right] n_x^{(i)} \delta(l_i) + \right.$$

$$+ \frac{\partial}{\partial x}\left(\left[\frac{\partial^2 w}{\partial x \partial y}\right] n_x^{(i)} \delta(l_i)\right) + \frac{\partial^2}{\partial x^2}\left(\left[\frac{\partial w}{\partial y}\right] n_x^{(i)} \delta(l_i)\right) +$$

$$\left. + \frac{\partial^3}{\partial x^3}\left([w] n_y^{(i)} \delta(l_i)\right)\right).$$

The second option.

$$\frac{\partial^4 w}{\partial x^3 \partial y} = \left\{\frac{\partial^4 w}{\partial x^3 \partial y}\right\} + \left\{\frac{\partial^3 w}{\partial x^2 \partial y}\right\} n_x^{(0)} \delta(l_0) +$$

$$+ \frac{\partial}{\partial x}\left(\left\{\frac{\partial^2 w}{\partial x \partial y}\right\} n_x^{(0)} \delta(l_0) + \frac{\partial^2}{\partial x^2}\left(\left\{\frac{\partial w}{\partial x}\right\} n_y^{(0)} \delta(l_0)\right) + \right.$$

$$+ \frac{\partial^3}{\partial x^2 \partial y}\left(w n_x^{(0)} \delta(l_0)\right) + \sum_{i=1}^{k}\left(\left[\frac{\partial^3 w}{\partial x^2 \partial y}\right] n_x^{(i)} \delta(l_i) + \right.$$

$$+ \frac{\partial}{\partial x}\left(\left[\frac{\partial^2 w}{\partial x \partial y}\right] n_x^{(i)} \delta(l_i)\right) + \frac{\partial^2}{\partial x^2}\left(\left[\frac{\partial w}{\partial x}\right] n_y^{(i)} \delta(l_i)\right) +$$

$$\left. + \frac{\partial^3}{\partial x^2 \partial y}\left([w] n_x^{(i)} \delta(l_i)\right)\right).$$

The third option.

$$\frac{\partial^4 w}{\partial x^3 \partial y} = \left\{\frac{\partial^4 w}{\partial x^3 \partial y}\right\} + \left\{\frac{\partial^3 w}{\partial x^2 \partial y}\right\} n_x^{(0)} \delta(l_0) +$$

$$+ \frac{\partial}{\partial x}\left(\left\{\frac{\partial^2 w}{\partial x^2}\right\} n_y^{(0)} \delta(l_0) + \frac{\partial^2}{\partial x \partial y}\left(\left\{\frac{\partial w}{\partial x}\right\} n_x^{(0)} \delta(l_0)\right) + \right.$$

$$\frac{\partial^3}{\partial x^2 \partial y}\left(w n_x^{(0)} \delta(l_0)\right) + \sum_{i=1}^{k}\left(\left[\frac{\partial^3 w}{\partial x^2 \partial y}\right] n_x^{(i)} \delta(l_i) + \right.$$

$$+\frac{\partial}{\partial x}\left(\left[\frac{\partial^2 w}{\partial x^2}\right]n_y^{(i)}\delta(l_i)\right)+\frac{\partial^2}{\partial x\partial y}\left(\left[\frac{\partial w}{\partial x}\right]n_x^{(i)}\delta(l_i)\right)+$$

$$+\frac{\partial^3}{\partial x^2\partial y}\Big([w]n_x^{(i)}\delta(l_i)\Big)\Big).$$

The fourth option.

$$\frac{\partial^4 w}{\partial x^3\partial y}=\left\{\frac{\partial^4 w}{\partial x^3\partial y}\right\}+\left\{\frac{\partial^3 w}{\partial x^3}\right\}n_y^{(0)}\delta(l_0)+$$

$$+\frac{\partial}{\partial y}\left(\left\{\frac{\partial^2 w}{\partial x^2}\right\}n_x^{(0)}\delta(l_0)+\frac{\partial^2}{\partial x\partial y}\left(\left\{\frac{\partial w}{\partial x}\right\}n_x^{(0)}\delta(l_0)\right)+\right.$$

$$+\frac{\partial^3}{\partial x^2\partial y}\Big(wn_x^{(0)}\delta(l_0)\Big)+\sum_{i=1}^{k}\left(\left[\frac{\partial^3 w}{\partial x^3}\right]n_y^{(i)}\delta(l_i)+\right.$$

$$+\frac{\partial}{\partial y}\left(\left[\frac{\partial^2 w}{\partial x^2}\right]n_x^{(i)}\delta(l_i)\right)+\frac{\partial^2}{\partial x\partial y}\left(\left[\frac{\partial w}{\partial x}\right]n_x^{(i)}\delta(l_i)\right)+$$

$$+\frac{\partial^3}{\partial x^2\partial y}\Big([w]n_x^{(i)}\delta(l_i)\Big)\Big).$$

Four options for $\frac{\partial^4 w}{\partial x\partial y^3}$ are obtained from $\frac{\partial^4 w}{\partial x^3\partial y}$ the permutation of letters x and y.

For $\frac{\partial^4 w}{\partial x^2\partial y^2}$, there are six options representations.

The first option.

$$\frac{\partial^4 w}{\partial x^2\partial y^2}=\left\{\frac{\partial^4 w}{\partial x^2\partial y^2}\right\}+\left\{\frac{\partial^3 w}{\partial x^2\partial y}\right\}n_y^{(0)}\delta(l_0)+$$

$$+\frac{\partial}{\partial y}\left(\left\{\frac{\partial^2 w}{\partial x^2}\right\}n_y^{(0)}\delta(l_0)+\frac{\partial^2}{\partial y^2}\left(\left\{\frac{\partial w}{\partial x}\right\}n_x^{(0)}\delta(l_0)\right)+\right.$$

$$+\frac{\partial^3}{\partial x\partial y^2}\Big(wn_x^{(0)}\delta(l_0)\Big)+\sum_{i=1}^{k}\left(\left[\frac{\partial^3 w}{\partial x^2\partial y}\right]n_y^{(i)}\delta(l_i)+\right.$$

$$+\frac{\partial}{\partial y}\left(\left[\frac{\partial^2 w}{\partial x^2}\right]n_y^{(i)}\delta(l_i)\right)+\frac{\partial^2}{\partial y^2}\left(\left[\frac{\partial w}{\partial x}\right]n_x^{(i)}\delta(l_i)\right)+$$

$$+\frac{\partial^3}{\partial x\partial y^2}\Big([w]n_x^{(i)}\delta(l_i)\Big)\Big).$$

The second option.

$$\frac{\partial^4 w}{\partial x^2\partial y^2}=\left\{\frac{\partial^4 w}{\partial x^2\partial y^2}\right\}+\left\{\frac{\partial^3 w}{\partial x\partial y^2}\right\}n_x^{(0)}\delta(l_0)+$$

$$+\frac{\partial}{\partial x}\left(\left\{\frac{\partial^2 w}{\partial y^2}\right\}n_x^{(0)}\delta(l_0)+\frac{\partial^2}{\partial x^2}\left(\left\{\frac{\partial w}{\partial y}\right\}n_y^{(0)}\delta(l_0)\right)+\right.$$

$$+\frac{\partial^3}{\partial x^2\partial y}\Big(wn_y^{(0)}\delta(l_0)\Big)+\sum_{i=1}^{k}\left(\left[\frac{\partial^3 w}{\partial x\partial y^2}\right]n_x^{(i)}\delta(l_i)+\right.$$

$$+\frac{\partial}{\partial x}\left(\left[\frac{\partial^2 w}{\partial y^2}\right]n_x^{(i)}\delta(l_i)\right)+\frac{\partial^2}{\partial x^2}\left(\left[\frac{\partial w}{\partial y}\right]n_y^{(i)}\delta(l_i)\right)+$$

$$+\frac{\partial^3}{\partial x^2\partial y}\Big([w]n_y^{(i)}\delta(l_i)\Big)\Big).$$

The third option.

$$\frac{\partial^4 w}{\partial x^2\partial y^2}=\left\{\frac{\partial^4 w}{\partial x^2\partial y^2}\right\}+\left\{\frac{\partial^3 w}{\partial x\partial y^2}\right\}n_x^{(0)}\delta(l_0)+$$

$$+\frac{\partial}{\partial x}\left(\left\{\frac{\partial^2 w}{\partial x\partial y}\right\}n_y^{(0)}\delta(l_0)+\frac{\partial^2}{\partial x\partial y}\left(\left\{\frac{\partial w}{\partial y}\right\}n_x^{(0)}\delta(l_0)\right)+\right.$$

$$+\frac{\partial^3}{\partial x^2\partial y}\Big(wn_y^{(0)}\delta(l_0)\Big)+\sum_{i=1}^{k}\left(\left[\frac{\partial^3 w}{\partial x\partial y^2}\right]n_x^{(i)}\delta(l_i)+\right.$$

$$+\frac{\partial}{\partial x}\left(\left[\frac{\partial^2 w}{\partial x\partial y}\right]n_y^{(i)}\delta(l_i)\right)+\frac{\partial^2}{\partial x\partial y}\left(\left[\frac{\partial w}{\partial y}\right]n_x^{(i)}\delta(l_i)\right)+$$

$$+\frac{\partial^3}{\partial x^2\partial y}\Big([w]n_y^{(i)}\delta(l_i)\Big)\Big).$$

The fourth option.

$$\frac{\partial^4 w}{\partial x^2\partial y^2}=\left\{\frac{\partial^4 w}{\partial x^2\partial y^2}\right\}+\left\{\frac{\partial^3 w}{\partial x^2\partial y}\right\}n_y^{(0)}\delta(l_0)+$$

$$+\frac{\partial}{\partial y}\left(\left\{\frac{\partial^2 w}{\partial x\partial y}\right\}n_x^{(0)}\delta(l_0)+\frac{\partial^2}{\partial x\partial y}\left(\left\{\frac{\partial w}{\partial x}\right\}n_y^{(0)}\delta(l_0)\right)+\right.$$

$$+\frac{\partial^3}{\partial x\partial y^2}\Big(wn_x^{(0)}\delta(l_0)\Big)+\sum_{i=1}^{k}\left(\left[\frac{\partial^3 w}{\partial x^2\partial y}\right]n_y^{(i)}\delta(l_i)+\right.$$

$$+\frac{\partial}{\partial y}\left(\left[\frac{\partial^2 w}{\partial x\partial y}\right]n_x^{(i)}\delta(l_i)\right)+\frac{\partial^2}{\partial x\partial y}\left(\left[\frac{\partial w}{\partial x}\right]n_y^{(i)}\delta(l_i)\right)+$$

$$+\frac{\partial^3}{\partial x\partial y^2}\Big([w]n_x^{(i)}\delta(l_i)\Big)\Big).$$

The fifth option.

$$\frac{\partial^4 w}{\partial x^2\partial y^2}=\left\{\frac{\partial^4 w}{\partial x^2\partial y^2}\right\}+\left\{\frac{\partial^3 w}{\partial x\partial y^2}\right\}n_x^{(0)}\delta(l_0)+$$

$$+\frac{\partial}{\partial x}\left(\left\{\frac{\partial^2 w}{\partial x\partial y}\right\}n_y^{(0)}\delta(l_0)+\frac{\partial^2}{\partial x\partial y}\left(\left\{\frac{\partial w}{\partial x}\right\}n_y^{(0)}\delta(l_0)\right)+\right.$$

$$+\frac{\partial^3}{\partial x\partial y^2}\left(wn_x^{(0)}\delta(l_0)\right)+\sum_{i=1}^{k}\left(\left[\frac{\partial^3 w}{\partial x\partial y^2}\right]n_x^{(i)}\delta(l_i)+\right.$$

$$+\frac{\partial}{\partial x}\left(\left[\frac{\partial^2 w}{\partial x\partial y}\right]n_y^{(i)}\delta(l_i)\right)+\frac{\partial^2}{\partial x\partial y}\left(\left[\frac{\partial w}{\partial x}\right]n_y^{(i)}\delta(l_i)\right)+$$

$$\left.+\frac{\partial^3}{\partial x\partial y^2}\left([w]n_x^{(i)}\delta(l_i)\right)\right).$$

The sixth option.

$$\frac{\partial^4 w}{\partial x^2\partial y^2}=\left\{\frac{\partial^4 w}{\partial x^2\partial y^2}\right\}+\left\{\frac{\partial^3 w}{\partial x^2\partial y}\right\}n_y^{(0)}\delta(l_0)+$$

$$+\frac{\partial}{\partial y}\left(\left\{\frac{\partial^2 w}{\partial x\partial y}\right\}n_x^{(0)}\delta(l_0)+\frac{\partial^2}{\partial x\partial y}\left(\left\{\frac{\partial w}{\partial y}\right\}n_x^{(0)}\delta(l_0)\right)+\right.$$

$$+\frac{\partial^3}{\partial x^2\partial y}\left(wn_y^{(0)}\delta(l_0)\right)+\sum_{i=1}^{k}\left(\left[\frac{\partial^3 w}{\partial x^2\partial y}\right]n_y^{(i)}\delta(l_i)+\right.$$

$$+\frac{\partial}{\partial y}\left(\left[\frac{\partial^2 w}{\partial x\partial y}\right]n_x^{(i)}\delta(l_i)\right)+\frac{\partial^2}{\partial x\partial y}\left(\left[\frac{\partial w}{\partial y}\right]n_x^{(i)}\delta(l_i)\right)+$$

$$\left.+\frac{\partial^3}{\partial x^2\partial y}\left([w]n_y^{(i)}\delta(l_i)\right)\right).$$

There $[g(x,y)]$ – jump function $g(x,y)$ when passing through a curve l_i in the selected direction of the normal; $\delta(l_i)$ – delta function concentrated on the curve l_i.

If $\varphi(x,y)\in C^{\infty}(R^2)$ – the basic function, then

$$\left\langle D^{\alpha}\mu(x,y)\delta(l),\,\varphi(x,y)\right\rangle=$$

$$=(-1)^{|\alpha|}\left\langle \mu(x,y)\delta(l),\,D^{\alpha}\varphi(x,y)\right\rangle=$$

$$=(-1)^{|\alpha|}\int_{l}\mu(x,y)\,D^{\alpha}\varphi(x,y)dl.$$

Substituting the expressions obtained in (1), we obtain

$$L(w)=D_{11}\frac{\partial^4 w}{\partial x^4}+4D_{16}\frac{\partial^4 w}{\partial x^3\partial y}+2(D_{12}+$$

$$+2D_{66})\frac{\partial^4 w}{\partial x^2\partial y^2}+4D_{26}\frac{\partial^4 w}{\partial x\partial y^3}+D_{22}\frac{\partial^4 w}{\partial y^4}=$$

$$=q(x,y)+\sum_{i=0}^{k}\left(I_1^{(i)}\delta(l_i)+\frac{\partial}{\partial x}\left(I_2^{(i)}\delta(l_i)\right)+\right.$$

$$+\frac{\partial}{\partial y}\left(I_3^{(i)}\delta(l_i)\right)+\frac{\partial^2}{\partial x^2}\left(I_4^{(i)}\delta(l_i)\right)+$$

$$+\frac{\partial^2}{\partial y^2}\left(I_5^{(i)}\delta(l_i)\right)+\frac{\partial^2}{\partial x\partial y}\left(I_6^{(i)}\delta(l_i)\right)+$$

$$+\frac{\partial^3}{\partial x^3}\left(I_7^{(i)}\delta(l_i)\right)+\frac{\partial^3}{\partial y^3}\left(I_8^{(i)}\delta(l_i)\right)+$$

$$\left.+\frac{\partial^3}{\partial x^2\partial y}\left(I_9^{(i)}\delta(l_i)\right)+\frac{\partial^3}{\partial x\partial y^2}\left(I_{10}^{(i)}\delta(l_i)\right)\right), \quad (2)$$

where

$$I_1^{(i)}=D_{11}\left[\frac{\partial^3 w}{\partial x^3}\right]n_x^{(i)}+D_{22}\left[\frac{\partial^3 w}{\partial y^3}\right]n_y^{(i)}+$$

$$+3D_{16}\left[\frac{\partial^3 w}{\partial x^2\partial y}\right]n_x^{(i)}+D_{16}\left[\frac{\partial^3 w}{\partial x^3}\right]n_y^{(i)}+$$

$$+3D_{26}\left[\frac{\partial^3 w}{\partial x\partial y^2}\right]n_y^{(i)}+D_{26}\left[\frac{\partial^3 w}{\partial y^3}\right]n_x^{(i)}+$$

$$+D_{12}\left[\frac{\partial^3 w}{\partial x^2\partial y}\right]n_y^{(i)}+D_{12}\left[\frac{\partial^3 w}{\partial x\partial y^2}\right]n_x^{(i)}+$$

$$+2D_{66}\left[\frac{\partial^3 w}{\partial x\partial y^2}\right]n_x^{(i)}+2D_{66}\left[\frac{\partial^3 w}{\partial x^2\partial y}\right]n_y^{(i)};$$

$$I_2^{(i)}=D_{11}\left[\frac{\partial^2 w}{\partial x^2}\right]n_x^{(i)}+2D_{16}\left[\frac{\partial^2 w}{\partial x\partial y}\right]n_x^{(i)}+$$

$$+D_{16}\left[\frac{\partial^2 w}{\partial x^2}\right]n_y^{(i)}+D_{26}\left[\frac{\partial^2 w}{\partial y^2}\right]n_y^{(i)}+$$

$$+D_{12}\left[\frac{\partial^2 w}{\partial y^2}\right]n_x^{(i)}+2D_{66}\left[\frac{\partial^2 w}{\partial x\partial y}\right]n_y^{(i)};$$

$$I_3^{(i)}=D_{22}\left[\frac{\partial^2 w}{\partial y^2}\right]n_y^{(i)}+D_{16}\left[\frac{\partial^2 w}{\partial x^2}\right]n_x^{(i)}+$$

$$+2D_{26}\left[\frac{\partial^2 w}{\partial x\partial y}\right]n_y^{(i)}+D_{26}\left[\frac{\partial^2 w}{\partial y^2}\right]n_x^{(i)}+$$

$$+D_{12}\left[\frac{\partial^2 w}{\partial x^2}\right]n_y^{(i)}+2D_{66}\left[\frac{\partial^2 w}{\partial x\partial y}\right]n_x^{(i)};$$

$$I_4^{(i)}=D_{11}\left[\frac{\partial w}{\partial x}\right]n_x^{(i)}+D_{16}\left[\frac{\partial w}{\partial y}\right]n_x^{(i)}+$$

$$+D_{16}\left[\frac{\partial w}{\partial x}\right]n_y^{(i)}+D_{12}\left[\frac{\partial w}{\partial y}\right]n_y^{(i)};$$

$$I_5^{(i)} = D_{22}\left[\frac{\partial w}{\partial y}\right]n_y^{(i)} + D_{26}\left[\frac{\partial w}{\partial x}\right]n_y^{(i)} +$$

$$+ D_{26}\left[\frac{\partial w}{\partial y}\right]n_x^{(i)} + D_{12}\left[\frac{\partial w}{\partial x}\right]n_x^{(i)};$$

$$I_6^{(i)} = 2D_{16}\left[\frac{\partial w}{\partial x}\right]n_x^{(i)} + 2D_{26}\left[\frac{\partial w}{\partial y}\right]n_y^{(i)} +$$

$$+ 2D_{66}\left[\frac{\partial w}{\partial y}\right]n_x^{(i)} + 2D_{66}\left[\frac{\partial w}{\partial x}\right]n_y^{(i)};$$

$$I_7^{(i)} = D_{11}[w]n_x^{(i)} + D_{16}[w]n_y^{(i)};$$

$$I_8^{(i)} = D_{22}[w]n_y^{(i)} + D_{26}[w]n_x^{(i)};$$

$$I_9^{(i)} = 3D_{16}[w]n_x^{(i)} + D_{12}[w]n_y^{(i)} + 2D_{66}[w]n_y^{(i)};$$

$$I_{10}^{(i)} = 3D_{26}[w]n_y^{(i)} + D_{12}[w]n_x^{(i)} + 2D_{66}[w]n_x^{(i)}.$$

Multiply $I_4^{(i)}$, $I_5^{(i)}$, $I_6^{(i)}$ ($i = \overline{0,k}$) on $(n_x^{(i)})^2 + (n_y^{(i)})^2 = 1$.

After transformations we get

$$L(w) = q(x,y) + \sum_{i=0}^{k}\Big(-\left[N_{n_i}\right]\delta(l_i) -$$

$$-\frac{\partial^2}{\partial x^2}\left(\left[\frac{\partial w}{\partial n_i}\right]\Big(D_{11}(n_x^{(i)})^2 + D_{12}(n_y^{(i)})^2 +\right.$$

$$+ 2D_{16}n_x^{(i)}n_y^{(i)})\delta(l_i) - \frac{\partial^2}{\partial y^2}\left(\left[\frac{\partial w}{\partial n^{(i)}}\right]\Big(D_{12}(n_x^{(i)})^2 +\right.$$

$$+ D_{22}(n_y^{(i)})^2 + 2D_{66}n_x^{(i)}n_y^{(i)})\delta(l_i) -$$

$$-2\frac{\partial^2}{\partial x \partial y}\left(\left[\frac{\partial w}{\partial n^{(i)}}\right]\Big(D_{16}(n_x^{(i)})^2 + D_{26}(n_y^{(i)})^2 +\right.$$

$$+ 2D_{16}n_x^{(i)}n_y^{(i)})\delta(l_i)\Big) -$$

$$-\frac{\partial^2}{\partial x^2}\left(\left[\frac{\partial w}{\partial \tau^{(i)}}\right]\Big(D_{11}n_x^{(i)}n_y^{(i)} - D_{12}n_x^{(i)}n_y^{(i)} +\right.$$

$$+ D_{16}(n_y^{(i)})^2 - D_{16}(n_x^{(i)})^2\Big)\delta(l_i)\Big) -$$

$$-\frac{\partial^2}{\partial y^2}\left(\left[\frac{\partial w}{\partial \tau^{(i)}}\right]\Big(D_{12}n_x^{(i)}n_y^{(i)} - D_{22}n_x^{(i)}n_y^{(i)} +\right.$$

$$+ D_{26}(n_y^{(i)})^2 - D_{26}(n_x^{(i)})^2\Big)\delta(l_i)\Big) -$$

$$-2\frac{\partial^2}{\partial x \partial y}\left(\left[\frac{\partial w}{\partial \tau^{(i)}}\right]\Big(D_{16}n_x^{(i)}n_y^{(i)} - D_{26}n_x^{(i)}n_y^{(i)} +\right.$$

$$+ D_{66}(n_y^{(i)})^2 - D_{66}(n_x^{(i)})^2\Big)\delta(l_i)\Big) +$$

$$+ \frac{\partial}{\partial n^{(i)}}\Big(\left[M_{n_i}\right]\delta(l_i)\Big) - \frac{\partial}{\partial \tau^{(i)}}\Big(\left[H_{tn_i}\right]\delta(l_i)\Big) +$$

$$+ \frac{\partial^3}{\partial x^3}\Big([w]\Big(D_{11}n_x^{(i)} + D_{16}n_y^{(i)}\Big)\delta(l_i)\Big) +$$

$$+ \frac{\partial^3}{\partial y^3}\Big([w]\Big(D_{22}n_y^{(i)} + D_{26}n_x^{(i)}\Big)\delta(l_i)\Big) +$$

$$+ \frac{\partial^3}{\partial x^2 \partial y}\Big([w]\Big(3D_{16}n_x^{(i)} + D_{12}n_y^{(i)} + 2D_{66}n_y^{(i)}\Big)\delta(l_i)\Big) +$$

$$+ \frac{\partial^3}{\partial x \partial y^2}\Big([w]\Big(3D_{26}n_y^{(i)} + D_{12}n_x^{(i)} + 2D_{66}n_x^{(i)}\Big)\delta(l_i)\Big)\Big). \quad (3)$$

The solution of equation (3) is obtained as the convolution of the fundamental solution of the operator $L(w)$ with the right part (3).

The fundamental solution $\Gamma(x,y)$ of the operator $L(w)$ is a solution of equation

$$L\big(\Gamma(x,y)\big) = \delta(x)\delta(y). \quad (4)$$

This solution was obtained in [6] and has two options. The first option corresponds to a case when equation $D_{11}\mu^4 + 4D_{16}\mu^3 + 2(D_{12} + 2D_{66})\mu^2 + +4D_{26}\mu + D_{22} = 0$ has four different roots:

$\mu_{1,2} = \alpha_1 \pm i\beta_1$, $\mu_{3,4} = \alpha_2 \pm i\beta_2$, $\alpha_i \in R$, $\beta_i > 0$ ($i = 1,2$).

The second option is obtained when the reduced equation has multiple roots

$\mu_{1,2} = \alpha + i\beta$, $\mu_{3,4} = \alpha - i\beta$, $\alpha \in R$, $\beta > 0$.

The fundamental solution for the first variant of the roots has the form:

$$\Gamma(x,y) = \frac{1}{4\pi\beta_1\beta_2 Q D_{11}}\Big\{\beta_2(\alpha_1^2 + \alpha_2^2 + \beta_2^2 - \beta_1^2 -$$

$$- 2\alpha_1\alpha_2)\left[\overline{r_1}^2 \ln r_1 + 2\beta_1 x(y + \alpha_1 x)\theta_1\right] +$$

$$+ \beta_1(\alpha_1^2 + \alpha_2^2 + \beta_1^2 - \beta_2^2 - 2\alpha_1\alpha_2)\left[\overline{r_2}^2 \ln r_2 +\right.$$

$$\left. + 2\beta_2 x(y + \alpha_2 x)\theta_2\right] + 2\beta_1\beta_2(\alpha_2 - \alpha_1) \times$$

$$\times\left[\overline{r_1}^2\theta_1 - 2\beta_1 x(y + \alpha_1 x)\ln r_1 -\right.$$

$$\left. - \overline{r_2}^2\theta_2 + 2\beta_2 x(y + \alpha_2 x)\ln r_2\right]\Big\}, \quad (5)$$

where

$$r_i = (x^2(\alpha_i^2 + \beta_i^2) + 2\alpha_i xy + y^2)^{1/2},$$

$$\bar{r}_i = (x^2(\alpha_i^2 - \beta_i^2) + 2\alpha_i xy + y^2)^{1/2},$$

$$\theta_i = arctg\left(\frac{y}{x\beta_i} + \frac{\alpha_i}{\beta_i}\right),$$

$$Q = \beta_1^4 + \beta_2^4 + \alpha_1^4 + \alpha_2^4 + 6\alpha_1^2\alpha_2^2 - 2\beta_1^2\beta_2^2 +$$

$$+ 2\beta_1^2\alpha_1^2 + 2\beta_1^2\alpha_2^2 - 4\beta_1^2\alpha_1\alpha_2 + 2\beta_1^2\alpha_1^2 +$$

$$+ 2\beta_2^2\alpha_2^2 - 4\beta_2^2\alpha_1\alpha_2 - 4\alpha_1^3\alpha_2 - 4\alpha_1\alpha_2^3.$$

For the second variant of the roots

$$\Gamma(x,y) = \frac{1}{8\pi\beta^3 D_{11}}\left(x^2(\alpha^2+\beta^2) + 2\alpha xy + y^2\right)\times$$

$$\times \ln\left(x^2(\alpha^2+\beta^2) + 2\alpha xy + y^2\right)^{1/2}. \qquad (6)$$

Thus,

$$\Gamma(x,y) * D^{\alpha}_{(x,y)}(\mu(x,y)\delta(l)) =$$

$$= \int_l D^{\alpha}_{(x,y)}(\Gamma(x-\xi, y-\eta))\mu(\xi,\eta)ds_{(\xi,\eta)} =$$

$$= (-1)^{|\alpha|}\int_l D^{\alpha}_{(\xi,\eta)}(\Gamma(x-\xi, y-\eta))\mu(\xi,\eta)ds_{(\xi,\eta)}.$$

Denoting, $G(x,y,\xi,\eta) = \Gamma(x-\xi, y-\eta)$, we get

Denoting, $G(x,y,\xi,\eta) = \Gamma(x-\xi, y-\eta)$, we get

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$$= (-1)^{|\alpha|}\int_l D^{\alpha}_{(\xi,\eta)}(G(x,y,\xi,\eta)\mu(\xi,\eta)ds_{(\xi,\eta)}. \qquad (7)$$

Thus, the solution of equation (3) has the form:

$$W(x,y) = \iint_B G(x,y,\xi,\eta)q(\xi,\eta)d\xi d\eta +$$

$$+ \sum_{i=0}^{k}\left(-\int_{l_i}\left[N_{n^{(i)}}\right]_{(\xi,\eta)} G(x,y,\xi,\eta)ds_{(\xi,\eta)} -\right.$$

$$-\int_{l_i}\left[\frac{\partial w}{\partial n^{(i)}}\right]_{(\xi,\eta)} (D_{11}(n_\xi^{(i)})^2 + D_{12}(n_\eta^{(i)})^2 +$$

$$+ 2D_{66}n_\xi^{(i)}n_\eta^{(i)})\frac{\partial^2 G(x,y,\xi,\eta)}{\partial\xi^2}ds_{(\xi,\eta)} -$$

$$-\int_{l_i}\left[\frac{\partial w}{\partial n^{(i)}}\right]_{(\xi,\eta)} (D_{12}(n_\xi^{(i)})^2 + D_{22}(n_\eta^{(i)})^2 +$$

$$+ 2D_{66}n_\xi^{(i)}n_\eta^{(i)})\frac{\partial^2 G(x,y,\xi,\eta)}{\partial\eta^2}ds_{(\xi,\eta)} -$$

$$-2\int_{l_i}\left[\frac{\partial w}{\partial n^{(i)}}\right]_{(\xi,\eta)} (D_{16}(n_\xi^{(i)})^2 + D_{26}(n_\eta^{(i)})^2 +$$

$$+ 2D_{66}n_\xi^{(i)}n_\eta^{(i)})\frac{\partial^2 G(x,y,\xi,\eta)}{\partial\xi\partial\eta}ds_{(\xi,\eta)} -$$

$$-\int_{l_i}\left[\frac{\partial w}{\partial \tau^{(i)}}\right]_{(\xi,\eta)} ((D_{11} - D_{12})n_\xi^{(i)}n_\eta^{(i)} +$$

$$+ D_{16}((n_\eta^{(i)})^2 - (n_\xi^{(i)})^2))\frac{\partial^2 G(x,y,\xi,\eta)}{\partial\xi^2}ds_{(\xi,\eta)} -$$

$$-\int_{l_i}\left[\frac{\partial w}{\partial \tau^{(i)}}\right]_{(\xi,\eta)} ((D_{12} - D_{22})n_\xi^{(i)}n_\eta^{(i)} +$$

$$+ D_{26}((n_\eta^{(i)})^2 - (n_\xi^{(i)})^2))\frac{\partial^2 G(x,y,\xi,\eta)}{\partial\eta^2}ds_{(\xi,\eta)} -$$

$$-2\int_{l_i}\left[\frac{\partial w}{\partial \tau^{(i)}}\right]_{(\xi,\eta)} ((D_{16} - D_{26})n_\xi^{(i)}n_\eta^{(i)} +$$

$$+ D_{66}((n_\eta^{(i)})^2 - (n_\xi^{(i)})^2))\frac{\partial^2 G(x,y,\xi,\eta)}{\partial\xi\partial\eta}ds_{(\xi,\eta)} -$$

$$-\int_{l_i}\left[M_{n_i}\right]_{(\xi,\eta)}\frac{\partial G(x,y,\xi,\eta)}{\partial n^{(i)}(\xi,\eta)}ds_{(\xi,\eta)} +$$

$$+\int_{l_i}\left[H_{tn^{(i)}}\right]_{(\xi,\eta)}\frac{\partial G(x,y,\xi,\eta)}{\partial \tau^{(i)}(\xi,\eta)}ds_{(\xi,\eta)} -$$

$$-\int_{l_i}[w]_{(\xi,\eta)}(D_{11}n_\xi^{(i)} + D_{16}n_\eta^{(i)})\frac{\partial^3 G(x,y,\xi,\eta)}{\partial\xi^3}ds_{(\xi,\eta)} -$$

$$-\int_{l_i}[w]_{(\xi,\eta)}(D_{22}n_\eta^{(i)} + D_{26}n_\xi^{(i)})\frac{\partial^3 G(x,y,\xi,\eta)}{\partial\eta^3}ds_{(\xi,\eta)} -$$

$$-\int_{l_i}[w]_{(\xi,\eta)}(3D_{16}n_\xi^{(i)} + (D_{12} + 2D_{66})n_\eta^{(i)})\times$$

$$\times\frac{\partial^3 G(x,y,\xi,\eta)}{\partial\xi^2\partial\eta}ds_{(\xi,\eta)} - \int_{l_i}[w]_{(\xi,\eta)}(3D_{26}n_\eta^{(i)} +$$

$$\left.+ (D_{12} + 2D_{66})n_\xi^{(i)})\frac{\partial^3 G(x,y,\xi,\eta)}{\partial\xi\partial\eta^2}ds_{(\xi,\eta)}\right). \qquad (8)$$

$G(x,y,\xi,\eta)$ can be considered as a deflection at a point (ξ,η) when the unit load concentrated at a point (x,y).

Denote $M_n(G(x,y,\xi,\eta))_{(\xi,\eta)}$ the bending moment corresponding deflection $G(\xi,\eta)$. Similarly, denote the remaining bending characteristics. After the transformation in (8) we obtain

$$W(x,y) = \iint_B G(x,y,\xi,\eta)d\xi d\eta +$$

$$+\sum_{i=0}^{k}\left(-\int_{l_i}\left[N_{n^{(i)}}\right]_{(\xi,\eta)}G(x,y,\xi,\eta)ds_{(\xi,\eta)}+\right.$$

$$+\int_{l_i}\left[\frac{\partial w(\xi,\eta)}{\partial\eta^{(i)}}\right]_{(\xi,\eta)}M_{n^{(i)}}(G(x,y,\xi,\eta))_{(\xi,\eta)}ds_{(\xi,\eta)}-$$

$$-\int_{l_i}\left[\frac{\partial w(\xi,\eta)}{\partial\tau^{(i)}}\right]_{(\xi,\eta)}H_{tn^{(i)}}(G(x,y,\xi,\eta))_{(\xi,\eta)}ds_{(\xi,\eta)}-$$

$$-\int_{l_i}[M_n(\xi,\eta)]\frac{\partial G(x,y,\xi,\eta)}{\partial n^{(i)}(\xi,\eta)}ds_{(\xi,\eta)}+$$

$$+\int_{l_i}[H_{tn}(\xi,\eta)]\frac{\partial G(x,y,\xi,\eta)}{\partial\tau^{(i)}(\xi,\eta)}ds_{(\xi,\eta)}+$$

$$\left.-\int_{l_i}[w(\xi,\eta)]N_{n^i}(G(x,y,\xi,\eta))_{(\xi,\eta)}ds_{(\xi,\eta)}\right). \qquad (9)$$

Let $A_i(a_{i1},a_{i2})$, $B_i(b_{i1},b_{i2})$ ($i=\overline{1,k}$), $T_j(t_{j1},t_{j2})$ – corner points l_0.

$$\int_{A_iB_i}\left[\frac{\partial w(\xi,\eta)}{\partial\tau^{(i)}}\right]H_{tn^{(i)}}(G(x,y,\xi,\eta))ds_{(\xi,\eta)}=$$

$$=w(\xi(s_2),\eta(s_2)+0)H_{tn^{(i)}}(G(x,y,\xi(s_2),\eta(s_2))-$$

$$-w(\xi(s_1),\eta(s_1)+0)\ H_{tn^{(i)}}(G(x,y,\xi(s_1),\eta(s_1))-$$

$$-\int_{S_1}^{S_2}w(\xi(s),\eta(s)+0)\frac{dH_{tn^{(i)}}(G(x,y,\xi(s),\eta(s))}{ds}ds-$$

$$-w(\xi(s_2),\eta(s_2)-0)H_{tn^{(i)}}(G(x,y,\xi(s_2),\eta(s_2))+$$

$$+w(\xi(s_1),\eta(s_1)-0)H_{tn^{(i)}}(G(x,y,\xi(s_1),\eta(s_1))+$$

$$+\int_{S_1}^{S_2}w(\xi(s),\eta(s)-0)\frac{dH_{tn^{(i)}}(G(x,y,\xi(s),\eta(s))}{ds}ds=$$

$$=[w(b_{i1},b_{i2})]H_{tn^{(i)}}(G(x,y,b_{i1},b_{i2}))-$$

$$-[w(a_{i1},a_{i2})]H_{tn^{(i)}}(G(x,y,a_{i1},a_{i2}))-$$

$$-\int_{A_iB_i}[w(\xi,\eta)]\frac{\partial H_{tn^{(i)}}(G(x,y,\xi,\eta))}{\partial s(\xi,\eta)}ds_{(\xi,\eta)}.$$

$$\int_{A_iB_i}\left[H_{tn^{(i)}}(\xi,\eta)\right]\frac{\partial G(x,y,\xi,\eta)}{\partial\tau^{(i)}_{(\xi,\eta)}}ds_{(\xi,\eta)}=$$

$$=\left[H_{tn^{(i)}}w(b_{i1},b_{i2})\right]G(x,y,b_{i1},b_{i2})-$$

$$-\left[H_{tn^{(i)}}w(a_{i1},a_{i2})\right]G(x,y,a_{i1},a_{i2})-$$

$$-\int_{A_iB_i}\left[\frac{\partial H_{tn^{(i)}}(\xi,\eta)}{\partial s}\right]G(x,y,\xi,\eta)ds_{(\xi,\eta)},$$

$i=\overline{1,k}$.

Similar transformations could be done for the curve l_0 at the points $T_j(t_{j1},t_{j2})$, $j=\overline{1,m}$.

Given the expression for $Q_{n^{(i)}}$, we get

$$W(x,y)=\iint_B G(x,y,\xi,\eta)q(\xi,\eta)d\xi d\eta+$$

$$+\sum_{i=1}^{k}\left(-\int_{l_i}\left[Q_{n^{(i)}}(\xi,\eta)\right]_{(\xi,\eta)}G(x,y,\xi,\eta)ds_{(\xi,\eta)}+\right.$$

$$+\int_{l_i}\left[\frac{\partial w(\xi,\eta)}{\partial\eta^{(i)}}\right]_{(\xi,\eta)}M_{n^{(i)}}(G(x,y,\xi,\eta))_{(\xi,\eta)}ds_{(\xi,\eta)}-$$

$$-\int_{l_i}\left[M_{n^{(i)}}(\xi,\eta)\right]\frac{\partial G(x,y,\xi,\eta)}{\partial\eta^{(i)}_{(\xi,\eta)}}ds_{(\xi,\eta)}+$$

$$+\int_{l_i}[w(\xi,\eta)]Q_{n^{(i)}}(G(x,y,\xi,\eta))ds_{(\xi,\eta)}+$$

$$+[w(a_{i1},a_{i2})]H_{tn^{(i)}}(G(x,y,a_{i1},a_{i2}))-$$

$$-[w(b_{i1},b_{i2})]H_{tn^{(i)}}(G(x,y,b_{i1},b_{i2}))+$$

$$+\left[H_{tn^{(i)}}(b_{i1},b_{i2})\right]G(x,y,b_{i1},b_{i2})-$$

$$-\left[H_{tn^{(i)}}(a_{i1},a_{i2})\right]G(x,y,a_{i1},a_{i2}))+$$

$$+\int_{l_0}Q_{n^{(0)}}G(x,y,\xi,\eta)ds_{(\xi,\eta)}-$$

$$-\int_{l_0}\frac{\partial w(\xi,\eta)}{\partial n}M_{n^{(0)}}(G(x,y,\xi,\eta))ds_{(\xi,\eta)}+$$

$$+\int_{l_0}M_{n^{(0)}}(\xi,\eta)\frac{\partial G(x,y,\xi,\eta)}{\partial n^{(0)}_{(\xi,\eta)}}ds_{(\xi,\eta)}+$$

$$+\int_{l_0}w(\xi,\eta)Q_{n^{(0)}}(G(x,y,\xi,\eta))ds_{(\xi,\eta)}+$$

$$+\sum_{j=1}^{m}\Big(w(t_{j1},t_{j2})(H_{tn^{(01)}}(G(x,y,t_{j1},t_{j2}))-$$

$$-(H_{tn^{(02)}}(G(x,y,t_{j1},t_{j2}))-$$

$$-G(x,y,t_{j1},t_{j2})(H_{tn^{(01)}}(t_{j1},t_{j2})-H_{tn^{(02)}}(t_{j1},t_{j2}))),$$

there $n^{(01)}$ and $n^{(02)}$ – normal direction at the point $T_j(t_{j1},t_{j2})$, corresponding to the beginning and end of a bypass l_0 from T_j in T_j ($j=\overline{1,m}$). If the curve $l_i=A_iB_i$ has no points in common with l_k ($k\neq i$), then as a result of continuous jumps on the l_i outintegrated terms at the points A_i and B_i equals zero. From four integrals at l_i usually unknown jumps are contained in two. If l_i models the crack, the unknowns are the jumps of deflection and the normal angle of inclination. If l_i simulates the thin insert rigidly engaged with the plate, the unknown jumps moment and generalized shear force. Knowing the two boundary conditions on l_0 and two conditions on $l_i\,(i=\overline{1,k})$, to find the unknown functions can be obtained the system of boundary integral equations possibly strongly singular.

Conclusions

Obtained an integral representation for the deflection of the anisotropic plate containing defects (curves on which the discontinuities of the first kind: deflections, tilt angles, moments or generalized shear forces). The resulting representation allows us to reduce the boundary value problem of the bending to a system of integral equations.

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Левада В.С., Хижняк В.К., Левицька Т.І. Інтегральне подання розривного розв’язка задачі згину анізотропної пластини

Опираючись на співвідношення, що зв’язують похідні прогину, як узагальненої функції, зі звичайними похідними, одержали диференціальне рівняння, у правій частині якого містяться узагальнені функції, що мають стрибки прогину, кутів нахилу, моментів і узагальнених перерізних сил. Розв’язок рівняння отримано у вигляді згортки фундаментального розв’язку із правою частиною. Зі знайденого подання можуть бути отримані граничні інтегральні рівняння (ГІР) для розв’язання поставленої задачі. Ці ГІР можуть вирішуватися методом граничних елементів.

Ключові слова: *згин, анізотропна пластина, дефекти, розривний розв’язок, узагальнена функція, крайова задача.*

Левада В.С., Хижняк В.К., Левицкая Т.И. Интегральное представление разрывного решения задачи изгиба анизотропной пластины

Опираясь на соотношения, связывающие производные прогиба, как обобщенной функции, с обычными производными, получили дифференциальное уравнение, в правой части которого содержатся обобщенные функции, имеющие скачки прогиба, углов наклона, моментов и обобщенных перерезывающих сил. Решение уравнения получено в виде свертки фундаментального решения с правой частью. Из найденного представления могут быть получены граничные интегральные уравнения (ГИУ) для решения поставленной задачи. Эти ГИУ могут решаться методом граничных элементов.

Ключевые слова: *изгиб, анизотропная пластина, дефекты, разрывное решение, обобщенная функция, краевая задача.*