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STRESS-DEFORMED STATE OF THE SHELL WITH A SMALL INITIAL DEFLECTION UNDER THE ACTION OF THE END LOAD

Purpose of work. *Construction of method for calculating the stress-strain state of cylindrical shell with small initial deflection, to which an end load is applied, using the method of characteristics. Comparison of the calculation results of the obtained model with the works of other authors in this area.*

Research methods. *For the calculation, the equations of motion of the Timoshenko type shell were used, taking into account both the shear deformation and inertia of rotation, and some nonlinear terms, to which the method of characteristics was applied. To obtain the equations of shell motion, the Hamilton-Ostrogradsky variational principle was used.*

Results *method is proposed for calculating the stress-strain state of a cylindrical shell with a small initial deflection using characteristics. Comparative analysis of the calculation results with research in this area by other authors, which showed the effectiveness of the proposed method.*

Scientific novelty. *The equations of the classical theory of shells, based on the Kirchhoff-Love hypotheses, which do not take into account the shear deformation and inertia of rotation, as well as linear equations of the Timoshenko type, have become widespread. In this work, a model of the stress-strain state of an axisymmetric shell with small initial deflections is constructed, taking into account both shear deformation and rotational inertia, and some nonlinear terms.*

Practical value. *The proposed method can be used to calculate the stress-strain state of structures in which thin shells are present as elements, taking into account small initial deflection. This method makes it possible to study the influence of the characteristics of the initial deflection on the stress-strain state of the entire structure.*

Key words: *shell, small deflection, end load, equations of motion, characteristics.*

Introduction

Questions related to the determination of the deformed and stressed state of elastic shells are urgent problems of mechanics. In particular, dynamic problems for various types of shell loading are of interest. By now, the equations of the classical theory of shells, based on the Kirchhoff-Love hypotheses, which do not take into account the shear deformation and rotational inertia have become widespread, as well as linear equations of the Timoshenko type. In this work, a model of the stress-strain state of an axisymmetric shell with small initial deflections is constructed, taking into account both shear deformation and rotational inertia, and some nonlinear terms.

In a linear formulation, unsteady waves in homogeneous shell structures were investigated in [1–2, etc.]. Nonlinear problems of deformation of shell systems with geometric imperfections were considered by V.S. Gudramovich [4]. Composite constructions in a nonlinear formulation were solved in [3, etc.].

Mathematical formulation of the problem and research results

Consider a semi-infinite cylindrical shell of circular cross-section with constant thickness h. The Ox axis is directed along the generating line by the middle of the shell surface, and the Oy axis is orthogonal to the Ox axis. We place the origin of coordinates at the end of the shell (Fig. 1).

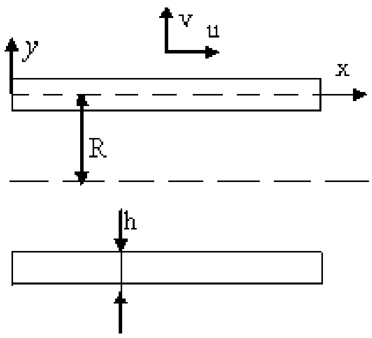

Fig. 1. Geometric interpretation of the problem statement

We will assume that the shell has a small initial deflection $\tilde{v}_0(x,y)$ in the middle surface. For the axisymmetric case: $\tilde{v}_0(x,y)=\tilde{v}_0(x)$, $\tilde{v}_1$ – additional elastic deflection, $\bar{v}=\bar{v}_0+\bar{v}_1$ – full deflection. In this case, in the expressions for deformations, only the components caused by the displacement of the dynamic deflection

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change, and they take the form:

$$\begin{cases} e_{xx} = \dfrac{\partial \tilde{u}}{\partial x} + \dfrac{1}{2}\left(\dfrac{\partial(\tilde{v}_0 + \tilde{v}_1)}{\partial x}\right)^2 - \dfrac{1}{2}\left(\dfrac{\partial \tilde{v}_0}{\partial x}\right)^2 \\ e_{\varphi\varphi} = \dfrac{\tilde{v}_1}{R+y} + \dfrac{\tilde{v}_1^{\,2}}{2(R+y)^2} \\ e_{xy} = \dfrac{1}{2}\left(\dfrac{\partial(\tilde{v}_1 + \tilde{v}_0)}{\partial x} + \dfrac{\partial \tilde{u}}{\partial y}\right) + \dfrac{1}{2}\dfrac{\partial(\tilde{v}_1 + \tilde{v}_0)}{\partial x} \cdot \dfrac{\partial(\tilde{v}_1 + \tilde{v}_0)}{\partial y} - \\ -\dfrac{1}{2}\dfrac{\partial \tilde{v}_0}{\partial x} \cdot \dfrac{\partial \tilde{v}_0}{\partial y} - \dfrac{1}{2}\dfrac{\partial \tilde{v}_0}{\partial x}. \end{cases} \quad (1)$$

Here $\tilde{u}(x,y,t)$ – displacement along the generatrix of the shell; $\tilde{v}(x,y,t)$ – displacement along the normal to the shell; R – middle surface radius.

We find the stress tensor components from Hooke's law:

$$\begin{cases} \sigma_{xx} = \dfrac{E}{1-\nu^2}(e_{xx} + \nu \cdot e_{\varphi\varphi}) + \dfrac{\nu}{1-\nu} \cdot \sigma_{yy} \\ \sigma_{\varphi\varphi} = \dfrac{E}{1-\nu^2}(e_{\varphi\varphi} + \nu \cdot e_{xx}) + \dfrac{\nu}{1-\nu} \cdot \sigma_{yy} \\ \tau = \dfrac{E}{1+\nu} e_{xy}. \end{cases} \quad (2)$$

Here $\tau = \sigma_{xy}$, E – Young's modulus, ν – Poisson's ratio.

We will look for displacements in the shell in this form:

$$\begin{cases} \tilde{u}(x,y,t) = u(x,t) + (y - \dfrac{h^2}{12R})\psi(x,t) \\ \tilde{v}(x,y,t) = v(x,t) = v_1(x,t) + v_0(x,0), \end{cases} \quad (3)$$

where the functions $u(x,t)$ and $v(x,t)$ can be considered as the displacement of some cylindrical surface $y = \dfrac{h^2}{12R}$, and ψ is the angle of rotation of the normal to the middle surface.

Assuming (as is customary for thin shells) $\sigma_{yy} = 0$, introducing the notation: $c_2^2 = \dfrac{E}{\rho(1-\nu^2)}$, $c_1^2 = \dfrac{E}{2\rho(1+\nu)}$ (ρ is the shell density) and using (1)–(3), we get:

$$\begin{cases} e_{xx} = \dfrac{\partial u}{\partial x} + (y - \dfrac{h^2}{12R})\dfrac{\partial \psi}{\partial x} + \dfrac{1}{2}\left(\dfrac{\partial (v_1)}{\partial x}\right)^2 + \dfrac{\partial v_0}{\partial x} \cdot \dfrac{\partial v_1}{\partial x} \\ e_{\varphi\varphi} = \dfrac{v_1}{R+y} + \dfrac{v_1^2}{2(R+y)^2} \\ e_{xy} = \dfrac{1}{2}\dfrac{\partial v_1}{\partial x} + \dfrac{1}{2}\psi, \end{cases}$$

$$\begin{cases} \sigma_{xx} = \rho c_2^2 \left[\dfrac{\partial u}{\partial x} + (y - \dfrac{h^2}{12R})\dfrac{\partial \psi}{\partial x} + \dfrac{1}{2}\left(\dfrac{\partial v_1}{\partial x}\right)^2 + \dfrac{\partial v_0}{\partial x} \cdot \dfrac{\partial v_1}{\partial x}\right] + \\ + \rho c_2^2 \nu \left[\dfrac{v_1}{R+y} + \dfrac{v_1^2}{2(R+y)^2}\right] \\ \sigma_{\varphi\varphi} = \rho c_2^2 \nu \left[\dfrac{\partial u}{\partial x} + (y - \dfrac{h^2}{12R})\dfrac{\partial \psi}{\partial x} + \dfrac{1}{2}\left(\dfrac{\partial v_1}{\partial x}\right)^2 + \dfrac{\partial v_0}{\partial x} \cdot \dfrac{\partial v_1}{\partial x}\right] + \\ + \rho c_2^2 \left[\dfrac{v_1}{R+y} + \dfrac{v_1^2}{2(R+y)^2}\right] \\ \tau = \rho c_1^2 \left(\dfrac{\partial v_1}{\partial x} + \psi\right). \end{cases} \quad (4)$$

To derive the equations of motion for the shell, we use the Hamilton-Ostrogradsky variational principle:

$$\delta \int_{t_2}^{t_1} (K - \Pi) dt = 0, \quad (5)$$

where K is the kinetic energy of the shell; Π is the potential energy of deformation. For a cylindrical shell, using formulas (4), we get:

$$K = \frac{1}{2}\int_0^{2\pi}\int_0^{\infty}\int_{-h/2}^{h/2} \rho \left[\left(\frac{\partial \tilde{u}}{\partial t}\right)^2 + \left(\frac{\partial \tilde{v}}{\partial t}\right)^2\right](R+y)\,dy\,dx\,d\varphi =$$

$$= \pi\rho \int_0^{\infty} \left[\left(\frac{\partial u}{\partial t}\right)^2 Rh + \frac{h^3}{12}R\left(\frac{\partial \psi}{\partial t}\right)^2 - \frac{h^5}{144R}\left(\frac{\partial \psi}{\partial t}\right)^2 + \right.$$

$$\left. + Rh\left(\frac{\partial v}{\partial t}\right)^2\right] dx,$$

$$\Pi = \frac{1}{2}\int_0^{2\pi}\int_0^{\infty}\int_{-h/2}^{h/2} \left[\sigma_{xx}e_{xx} + \sigma_{\varphi\varphi}e_{\varphi\varphi} + 2\tau e_{xy}\right](R+y)\,dy\,dx\,d\varphi =$$

$$= \pi\rho \int_0^{\infty} \left\{ c_2^2 \left[2\nu v_1 h \left(\frac{\partial u}{\partial x} - \frac{h^2}{12R}\cdot\frac{\partial \psi}{\partial x} + \frac{1}{2}\left(\frac{\partial v_1}{\partial x}\right)^2 + \frac{\partial v_0}{\partial x}\cdot\frac{\partial v_1}{\partial x}\right) + \right.\right.$$

$$+ \nu v_1^2 \ln\left|\frac{2R+h}{2R-h}\right| \cdot \left(\frac{\partial u}{\partial x} - \frac{h^2}{12R}\frac{\partial \psi}{\partial x} + \frac{1}{2}\left(\frac{\partial v_1}{\partial x}\right)^2 + \frac{\partial v_0}{\partial x}\cdot\frac{\partial v_1}{\partial x}\right) +$$

$$+ \nu v_1^2 h \frac{\partial \psi}{\partial x} - \nu R v_1^2 \ln\left|\frac{2R+h}{2R-h}\right| \frac{\partial \psi}{\partial x} +$$

$$+ Rh\left(\frac{\partial u}{\partial x} - \frac{h^2}{12R}\frac{\partial \psi}{\partial x} + \frac{1}{2}\left(\frac{\partial v_1}{\partial x}\right)^2 + \frac{\partial v_0}{\partial x}\cdot\frac{\partial v_1}{\partial x}\right)^2 + \frac{Rh^3}{12}\left(\frac{\partial \psi}{\partial x}\right)^2 +$$

$$+ \frac{h^3}{6}\cdot\frac{\partial \psi}{\partial x}\cdot\left(\frac{\partial u}{\partial x} - \frac{h^2}{12R}\frac{\partial \psi}{\partial x} + \frac{1}{2}\left(\frac{\partial v_1}{\partial x}\right)^2 + \frac{\partial v_0}{\partial x}\cdot\frac{\partial v_1}{\partial x}\right) +$$

$$\left. + v_1^2 \ln\left|\frac{2R+h}{2R-h}\right| + \frac{4h}{4R^2 - h^2} v_1^3 + \frac{4 v_1^4 Rh}{(4R^2 - h^2)^2}\right] +$$

$$\left. + c_1^2 Rh \left(\frac{\partial v_1}{\partial x} + \psi\right)^2 \cdot k^2 \right\} dx. \quad (7)$$

The last term in the resulting equality is multiplied by a correcting multiplier k^2.

Substituting the obtained expressions for the potential and kinetic energy in (5), and taking into account that the variations of the functions $\delta u, \delta v_1, \delta\psi$ are independent quantities, we find three equations describing the axisymmetric motion of a cylindrical shell with an initial deflection.

$$\frac{\partial^2 u}{\partial t^2}=\frac{c_2^2\nu}{R}\cdot\frac{\partial v_1}{\partial x}+\frac{c_2^2\nu}{Rh}\cdot\ln\left|\frac{2R+h}{2R-h}\right|\cdot v_1\frac{\partial v_1}{\partial x}+$$
$$+c_2^2\cdot\frac{\partial^2 u}{\partial x^2}+c_2^2\frac{\partial v_1}{\partial x}\cdot\frac{\partial^2 v_1}{\partial x^2}+$$
$$+c_2^2\cdot\left(\frac{\partial^2 v_0}{\partial x^2}\cdot\frac{\partial v_1}{\partial x}+\frac{\partial^2 v_1}{\partial x^2}\cdot\frac{\partial v_0}{\partial x}\right);$$

$$\frac{\partial^2\psi}{\partial t^2}\left(\frac{h^3}{12}R-\frac{h^5}{144R}\right)=c_2^2\left(\frac{h^3}{12}R-\frac{h^5}{144R}\right)\frac{\partial^2\psi}{\partial x^2}-$$
$$-\left(c_2^2\nu h^3\frac{1}{12R}+c_1^2k^2hR\right)\frac{\partial v_1}{\partial x}+$$

$$+v_1\frac{\partial v_1}{\partial x}\cdot\left[c_2^2\nu\left(h-R\ln\left(\frac{2R+h}{2R-h}\right)\right)-c_2^2\nu\frac{h^2}{12R}\ln\left|\frac{2R+h}{2R-h}\right|\right]-$$
$$-c_1^2k^2hR\psi;$$
$$\frac{\partial^2 v_1}{\partial t^2}=c_1^2k^2\frac{\partial^2 v_1}{\partial x^2}+\frac{c_2^2\nu}{R}\cdot 2\cdot v_1\frac{\partial^2 v_1}{\partial x^2}-$$
$$-\frac{c_2^2}{Rh}\ln\left|\frac{2R+h}{2R-h}\right|v_1+\left(c_1^2k^2+\frac{c_2^2\nu h^2}{12R^2}\right)\frac{\partial\psi}{\partial x}-$$

$$-\frac{c_2^2\nu}{R}\cdot\frac{\partial u}{\partial x}-\frac{c_2^2\nu}{2R}\left(\frac{\partial v_1}{\partial x}\right)^2-\frac{6c_2^2v_1^2}{(4R^2-h^2)R}-$$
$$-\frac{c_2^2\nu}{R}\left(\frac{\partial v_0}{\partial x}\cdot\frac{\partial v_1}{\partial x}-v_1\frac{\partial^2 v_0}{\partial x}\right).\tag{8}$$

When deriving these relations, it was assumed that it was possible to neglect nonlinear terms containing the function u and its derivatives, as well as nonlinear terms that include the displacements v1 and v0 or their derivatives if the degree of these terms is higher than two.

Next, we expand the coefficients in the resulting system (8) by degrees of h / R and retain only the senior terms in the expansion, as a result we obtain the following Timoshenko-type equations for the shell under consideration:

$$\frac{\partial^2 u}{\partial t^2}=c_2^2\frac{\partial^2 u}{\partial x^2}+c_2^2\frac{\nu}{R}\cdot\frac{\partial v_1}{\partial x}+c_2^2\frac{\partial v_1}{\partial x}\cdot\frac{\partial^2 v_1}{\partial x^2}+$$
$$+\frac{c_2^2\cdot\nu}{R^2}v_1\frac{\partial v_1}{\partial x}+c_2^2\frac{\partial^2 v_0}{\partial x^2}\cdot\frac{\partial v_1}{\partial x}+c_2^2\frac{\partial^2 v_1}{\partial x^2}\cdot\frac{\partial v_0}{\partial x};$$

$$\frac{\partial^2\psi}{\partial t^2}=c_2^2\frac{\partial^2\psi}{\partial x^2}-\left(\frac{c_2^2\nu}{R^2}+\frac{c_1^2k^2\cdot 12}{h^2}\right)\frac{\partial v_1}{\partial x}-\frac{12c_1^2}{h^2}k^2\psi-$$
$$-\frac{c_2^2\nu}{R^3}\cdot v_1\frac{\partial v_1}{\partial x};$$
$$\frac{\partial^2 v_1}{\partial t^2}=c_1^2k^2\frac{\partial^2 v_1}{\partial x^2}+c_1^2k^2\frac{\partial\psi}{\partial x}-\frac{c_1^2}{R^2}v_1-$$
$$-\frac{c_2^2\nu}{R}\cdot\frac{\partial u}{\partial x}-\frac{c_2^2\nu}{2R}\left(\frac{\partial v_1}{\partial x}\right)^2+$$
$$+\frac{2c_2^2\nu}{R}v_1\frac{\partial^2 v_1}{\partial x^2}-\frac{3c_2^2}{2R^3}v_1^2-\frac{c_2^2\nu}{R}\cdot\frac{\partial v_0}{\partial x}.\tag{9}$$

They represent a hyperbolic system of equations for the dynamic state of the shell. It is of interest to construct a solution to the problem considered here using characteristic equations. To begin with, we pass in equations (9) to dimensionless variables by the formulas:

$$\bar t=\frac{c_2t}{R};\quad \bar x=\frac{x}{R};\quad U=\frac{u}{R};$$
$$V_1=\frac{v_1}{R};\ V_0=\frac{v_0}{R};\ c=\frac{c_1}{c_2};\ \chi=\frac{h}{R}.\tag{10}$$

Then system (9) is reduced to the following:

$$U_{tt}=U_{xx}+\left(V_{1x}+V_{0x}\right)\cdot V_{1xx}+F_1;$$
$$\psi_{tt}=\psi_{xx}+F_2;$$
$$V_{1tt}=\left(c^2k^2+2\nu V_1\right)V_{1xx}+F_3.\tag{11}$$

where for convenience the following notation is introduced:

$$F_1=\nu V_{1x}+\nu V_1\cdot V_{1x}+V_{0xx}\cdot V_{1x};$$
$$F_2=-\left(\nu+12\frac{k^2c}{\chi^2}\right)V_{1x}-\frac{12k^2c}{\chi^2}\psi-\nu V_1V_{1x};$$
$$F_3=c^2k^2\psi_x-V_1-\nu U_x-\frac{\nu}{2}\left(V_{1x}\right)^2-$$
$$-\frac{3}{2}V_1^2-\nu V_{0x}V_{1x}+\nu V_1V_{0xx}.\tag{12}$$

In addition, the continuity conditions are satisfied along any direction:

$$dU_x=U_{xt}dt+U_{xx}dx;$$
$$dV_{1x}=V_{1xt}dt+V_{1xx}dx;$$
$$d\psi_x=\psi_{xt}dt+\psi_{xx}dx;$$
$$dU_t=U_{tt}dt+U_{tx}dx;$$
$$dV_{1t}=V_{1tt}dt+V_{1tx}dx;$$
$$d\psi_t=\psi_{tt}dt+\psi_{tx}dx.\tag{13}$$

Getting rid of the mixed derivative in equations (13), they can be rewritten as:

$$U_{tt} = U_{xx}\left(\frac{dx}{dt}\right)^2 + \frac{d(U_t)}{dt} - \frac{d(U_x)dx}{(dt)^2};$$

$$\psi_{tt} = \psi_{xx}\left(\frac{dx}{dt}\right)^2 + \frac{d(\psi_t)}{dt} - \frac{d(\psi_x)dx}{(dt)^2};$$

$$V_{1tt} = V_{1xx}\left(\frac{dx}{dt}\right)^2 + \frac{d(V_{1t})}{dt} - \frac{d(V_{1x})dx}{(dt)^2}. \quad (14)$$

Let us consider the second equations of system (11), (14) separately, since they do not depend on $U_{xx}, U_{tt}, V_{1tt}, V_{1xx}$. Without them, if we substitute U_{tt} and V_{1tt} from (14) into equations (11) and introduce the following notation:

$$\gamma = V_{1x} + V_{0x};\quad \beta^2 = c^2k^2 + 2\nu V_1;$$

$$\Phi_1 = \nu V_{1x} + \nu V_1 \cdot V_{1x} + V_{0xx} \cdot V_{1x} - \frac{dU_t}{dt} + d(U_x)\frac{dx}{(dt)^2};$$

$$\Phi_3 = c^2k^2\psi_x - V_1 - \nu U_x - \frac{\nu}{2}(V_{1x})^2 - \frac{3}{2}V_1^2 - \nu V_{0x}V_{1x} + \nu V_1 V_{0xx} - \frac{dV_{1t}}{dt} + d(V_{1x})\frac{dx}{(dt)^2}. \quad (15)$$

we get a system of two equations for U_{xx} and V_{1xx}:

$$\begin{cases} \left[\left(\frac{dx}{dt}\right)^2 - 1\right]U_{xx} - \gamma \cdot V_{1xx} = \Phi_1; \\ \left[\left(\frac{dx}{dt}\right)^2 - \beta^2\right]V_{1xx} = \Phi_3. \end{cases} \quad (16)$$

For this system to be linearly dependent, the following relationships must be met:

$$\frac{\left(\frac{dx}{dt}\right)^2 - 1}{0} = \frac{-\gamma}{\left(\frac{dx}{dt}\right)^2 - \beta^2} = \frac{\Phi_1}{\Phi_3}, \quad (17)$$

from which we find the characteristics and ratios on them:

$$dx = \pm\beta dt;$$

$$F_3 \cdot dx \mp \beta \cdot dV_{1t} + \beta^2 dV_{1x} = 0;$$

$$dx = \pm dt;$$

$$-\gamma\left(F_3 dx \mp dV_{1t} + d(V_{1x})\right) = (1-\beta^2)\left(F_1 dx \mp dU_t + dU\right). \quad (18)$$

Now consider the system obtained from the second equations of relations (11), (14):

$$\begin{cases} \psi_{tt} = \psi_{xx} + F_2; \\ \psi_{tt} - \left(\frac{dx}{dt}\right)^2 \psi_{xx} = \frac{d\psi_t}{dt} - \frac{d\psi_x}{dt}\left(\frac{dx}{dt}\right). \end{cases} \quad (19)$$

Let us find the characteristics from the equality to zero of the determinant of the system – Δ, and the relations on them from the equality to zero Δ1:

$$\Delta = \begin{vmatrix} 1 & -1 \\ 1 & -\left(\frac{dx}{dt}\right)^2 \end{vmatrix} = 0;$$

$$\Delta_1 = \begin{vmatrix} 1 & F_2 \\ 1 & F_4 \end{vmatrix} = F_4 - F_2 = 0; \quad (20)$$

where $F_4 = \frac{d\psi_t}{dt} - \frac{d\psi_x}{dt}\left(\frac{dx}{dt}\right)$.

Therefore, on the characteristics $dx = \pm dt$, we obtain, in addition to (18), the following relations:

$$F_2 \cdot dt = d\psi_t \mp d\psi_x. \quad (21)$$

The initial conditions were assumed to be zero. The mechanical effect on the shell was modeled by setting the particle velocity at the end $\bar{x} = \bar{x_0}$ in the form: $U_0 = \bar{t}e^{1-\bar{t}}$. Numerical calculations were carried out at

$d\bar{x} = d\bar{t} = 0.001$, $k^2 = 0.87$, $\bar{x_0} = 0$, $E = 2.1 * 10^{11} H/m^2$, $\nu = \frac{1}{3}$, $\frac{h}{R} = 0.05$, by the method of characteristics, which is described in detail in [5–7]. The function plots show the distributions of velocities for different points in time:

$\bar{t} = 2.5$ (plot 1), $\bar{t} = 4$ (plot 2), $\bar{t} = 5.5$ (plot 3). Here Fig. 2 and Fig. 3 correspond $\bar{v_0} = 0$, and Fig. 4 and Fig. 5 – $\bar{v_0}(\bar{x}) = 0.01 \cdot \left(\bar{x}^2 - 6\bar{x} + 8\right)$ for $x \in [2; 4]$, $\bar{v_0}(\bar{x}) = 0$ for $x \notin [2; 4]$.

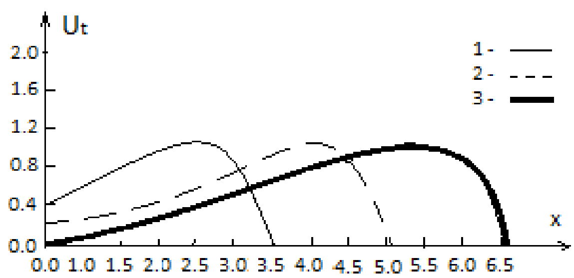

Fig. 2. Velocity distribution U_t at $\bar{v_0} = 0$

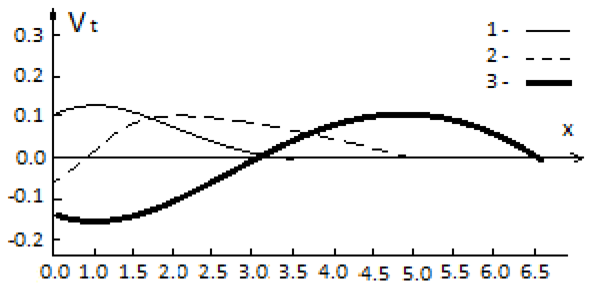

Fig. 3. Velocity distribution V_t at $\overline{v_0} = 0$

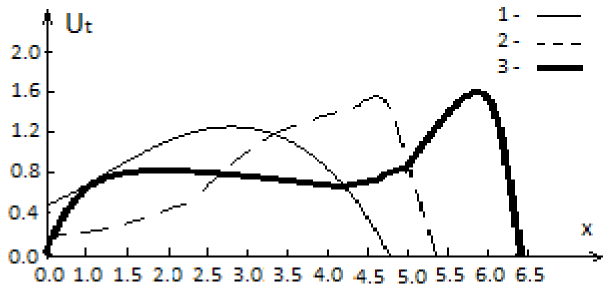

Fig. 4. Velocity distribution U_t at $\overline{v_0}(x)$

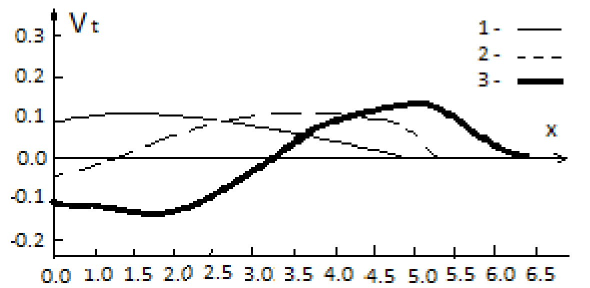

Fig. 5. Velocity distribution V_t at $\overline{v_0}(x)$

Conclusions

Analysis was carried out for various geometrical and physical parameters of the shell with a small initial deflection, as well as for various types and durations of end loading. In general, the results of this study are in good agreement with studies in this area by other authors, they show that the linear theory is quite acceptable when studying the transfer of a load impulse in a homogeneous shell and in a shell with a small initial deflection, the figures show the effect of nonzero deflection on the deformed state of the shell. In both cases, a rapid decay damping when moving away from the loading edge.

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Пожуєва І. С. Левицька Т. І. Напружено-деформований стан оболонки з малим початковим прогином під дією торцевого навантаження

Мета роботи. *Побудова методики розрахунку напружено-деформованого стану циліндричної оболонки з малим початковим прогином, до якої прикладається торцеве навантаження, з використанням методу характеристик. Зіставлення результатів розрахунку отриманої моделі з роботами інших авторів у цій області.*

Методи дослідження. *Для розрахунку було використано рівняння руху оболонки типу Тимошенко, що враховують як деформацію зсуву й інерцію обертання, так і деякі нелінійні члени, до яких був застосований метод характеристик. Для виводу рівнянь руху оболонки застосовувався варіаційний принцип Гамільтона-Остроградського.*

Отримані результати. *Запропоновано метод розрахунку напружено-деформованого стану циліндричної оболонки з малим початковим прогином за допомогою характеристик. Проведено порівняльний аналіз результатів розрахунків з дослідженнями в цій області інших авторів, що показав ефективність запропонованого методу.*

Наукова новизна. *Широке поширення одержали рівняння класичної теорії оболонок, засновані на гіпотезах Кірхгофа-Лява, що не враховують деформацію зсуву й інерцію обертання, а також лінійні рівняння типу*

Тимошенко. У даній роботі побудована модель напружено-деформованого стану осесиметричної оболонки з малими початковими прогинами, що враховує як деформацію зсуву й інерцію обертання, так і деякі нелінійні члени.

Практична цінність. *Запропонований метод може бути використаний для розрахунку напружено-деформованого стану конструкцій, у яких присутні у якості елементів тонкі оболонки з наявністю малого початкового прогину. Даний метод дозволяє проводити дослідження впливу характеристик початкового прогину на напружено-деформований стан всієї конструкції.*

Ключові слова: *оболонка, малий прогин, торцеве навантаження, рівняння руху, характеристики.*

Пожуева И. С., Левицкая Т. И. Напряженно-деформированное состояние оболочки с малым начальным прогибом под действием торцевой нагрузки

Цель работы. *Построение методики расчета напряженно-деформированного состояния цилиндрической оболочки с малым начальным прогибом, к которой прикладывается торцевая нагрузка, с использованием метода характеристик. Сопоставление результатов расчета полученной модели с работами других авторов в этой области.*

Методы исследования. *Для расчета использовались уравнения движения оболочки типа Тимошенко, учитывающие как деформацию сдвига и инерцию вращения, так и некоторые нелинейные члены, к которым был применен метод характеристик. Для вывода уравнений движения оболочки использовался вариационным принципом Гамильтона-Остроградского.*

Полученные результаты. *Предложен метод расчета напряженно-деформированного состояния цилиндрической оболочки с малым начальным прогибом с помощью характеристик. Проведен сравнительный анализ результатов расчетов с исследованиями в этой области других авторов, который показал эффективность предложенного метода.*

Научная новизна. *Широкое распространение получили уравнения классической теории оболочек, основанные на гипотезах Кирхгофа-Лява, не учитывающие деформацию сдвига и инерцию вращения, а также линейные уравнения типа Тимошенко. В данной работе построена модель напряженно-деформированного состояния осесимметричной оболочки с малыми начальными прогибами, учитывающая как деформацию сдвига и инерцию вращения, так и некоторые нелинейные члены.*

Практическая ценность. *Предложенный метод может быть использован для расчета напряженно-деформированного состояния конструкций, в которых присутствуют в качестве элементов тонкие оболочки с учетом малого начального прогиба. Данный метод позволяет проводить исследование влияния характеристик начального прогиба на напряженно-деформированное состояние всей конструкции.*

Ключевые слова: *оболочка, малый прогиб, торцевая нагрузка, уравнения движения, характеристики.*